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THERMODYNAMICS OF EFFECTIVE STRESS OF PARTIALLY SATURATED SOILS

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Summary. The effective stress of partially saturated soil is derived using a rigorous thermodynamic framework. The derivation considers the phenomenon of suction without the need to follow interfaces between solid, fluid and gas domains. Instead, suction is recovered via energy minimisation about a novel 'suctionless limit', while equilibrium of chemical potentials is ensured. It is shown that the effective stress is linked to the characteristics of soil-water retention curve (SWRC), and thus generally depends on the densities of the solid, fluid and gas domains. Special cases of SWRC without dependencies on gas and/or solid densities can then explain why previous empirical relations worked in particular loading and state conditions, but not in others.

1 Approach

Can an expression for the effective stress of partially saturated soil be defined? Can it capture soil response for all states and stress paths, for example under both shear failure and isotropic compression? The answers to these questions cannot be resolved empirically, but require a rigorous thermodynamic treatment. The current work pursues this strategy to unravel the general structure of effective stress. In soil mechanics the effective stress of soils is typically written as $\sigma_{ij}^{\text{eff}} = \sigma_{ij} - P_T \delta_{ij}$, with σ_{ij} being the total stress and P_T some sort of pressure. This is generally shown to be true for any degree of saturation, if σ_{ij}^{eff} and P_T are to be interpreted as the elastic stress and thermodynamic pressure.

1.1 Thermodynamic effective stress

It is further shown that the thermodynamic pressure can always be written in a Bishop form, $P_T = u_A + \chi\,(u_A - u_W)$ and that the measured suction $s = u_A - u_W$ and Bishop parameter χ are both generally dependent on the three thermodynamic densities of air, water and solid, ρ_A,ρ_W,ρ_S (where we used u_A and u_W for the air and water pressures in measurement cells). The relationships capturing these dependencies on densities could be written most generally as follows:

$$\sigma_{ij}^{\mathrm{eff}} = \sigma_{ij} - P_T \delta_{ij}, \quad P_T = u_A + \chi\,(u_A - u_W)\,, \tag{1}$$

$$\chi \equiv \chi\,(\rho_A, \rho_W, \rho_S) = \frac{\rho_W}{\hat{\rho}_W} + \frac{\rho_S}{\hat{\rho}_S}\left[\frac{\hat{\rho}_A \psi_A - \hat{\rho}_S \psi_S}{\hat{\rho}_A \psi_A - \hat{\rho}_W \psi_W}\right], \tag{2}$$

$$s \equiv s\,(\rho_A, \rho_W, \rho_S) = \frac{1}{2K_W}\frac{\rho_W}{\hat{\rho}_W}\left[\hat{\rho}_A \psi_A - \hat{\rho}_W \psi_W\right], \tag{3}$$

where hatted symbols refer to intrinsic properties; K_W refers to the water bulk modulus; $\hat{s} \equiv \hat{s}\,(\rho_A, \rho_W, \rho_S) = \hat{P}_A - \hat{P}_W = \sqrt{\psi}$ is the intrinsic suction (defined in terms of corresponding intrinsic pressures, $\hat{P}_A$ and $\hat{P}_W$); and we designate $\psi_\beta = \partial\psi/\partial\rho_\beta$, where index $\beta = \{A, W, S\}$ can be used for air, water or solid domains. Eqs. 1-3 give the parametric relationships between the effective stress and the characteristics of the measured suction, through the dependence of the intrinsic suction on the three thermodynamic densities. Assuming, as is usually done in soil mechanics, that the SWRC is only a function of the degree of saturation S_r and porosity n , Eqs. 2,3 reduce to:

$$\chi \equiv \chi\,(S_r, n) = S_r - n\,(1-n)\,\frac{\psi_n}{\psi_{Sr}}, \tag{4}$$

$$s \equiv s\,(S_r, n) = -\frac{S_r \psi_{Sr}}{2K_W}, \tag{5}$$

where $\psi_n = \partial\psi/\partial n$ and $\psi_{Sr} = \partial\psi/\partial S_r$. It is therefore shown that for SWRC independent on air density, χ should also be independent on air density, as frequently assumed in the literature. If the SWRC is further assumed to be independent on porosity (or voids ratio), such that $\psi_n = 0$, we recover the commonly used result $\chi = S_r$. However, any dependence on porosity introduces correction. The closure for the definition of the thermodynamic effective stress is given by postulating a functional form for the potential $\psi = \hat{s}^2$. Specifically, using an assumed form for the intrinsic suction $\hat{s} \equiv (S_r, n)$, and therefore an assumed ψ, we can define both the Bishop's parameter χ and the measured suction s based on Eqs. 4,5. The theoretical measured suction can then be validated using the experimental measured suction through the SWRC, to recover the true structure of the effective stress. Further details on this process will be provided in a future publication, and briefly presented during the meeting.

As mostly acknowledged in soil mechanics, the characteristics of the SWRC are generally known to be a function of both the degree of saturation and porosity, and therefore χ should also depend on porosity. The above equations therefore motivate to introduce the effect of porosity on the effective stress. Porosity tends to vary most significantly during compression, but not during shear failure, and this can potentially explain the difficulties of previous effective stress expressions in capturing soil behaviour under those two distinguishable conditions. For example, during isotropic compression, a change in porosity should affect the thermodynamic pressure according to Eq. 1. To understand this fact consider an isotropic loading followed by an unloading stage to the original total stress and degree of saturation, while the material porosity is let to change significantly. According to the formulation above the effective stress can vary appreciably due to the only change in porosity, which can justify previously unexplained swelling/contraction during such loading conditions.